Mental Math

3rd Annual WSMA Math Bowl
March 2, 2013
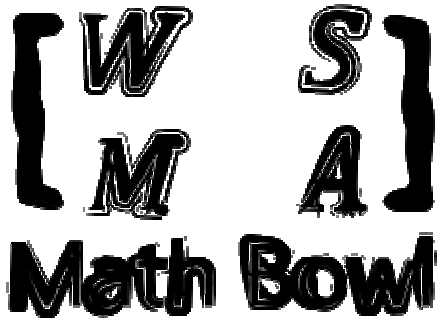
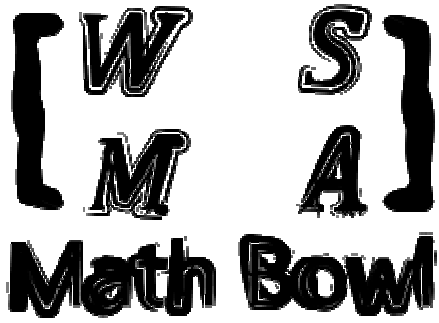# Problem 1

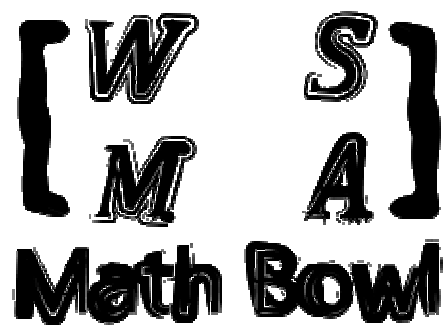How many multiples of 2 but not 3 are there from 1 to 1000 inclusive?
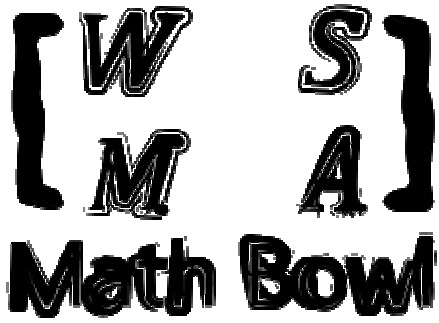
Find the units digit of 2^{81}.

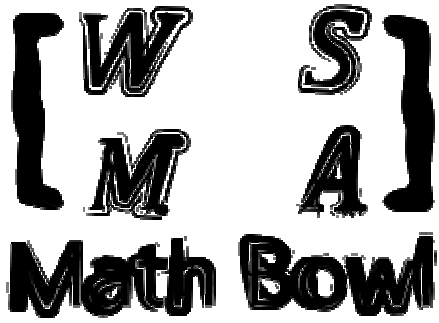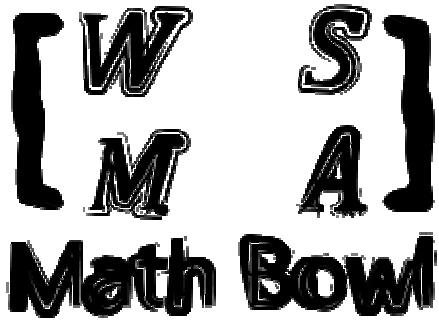# Problem 3

How many diagonals can be drawn in a dodecagon?

Problem 4

Find the fifth pentagonal number.
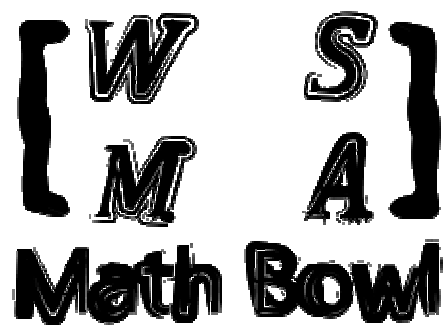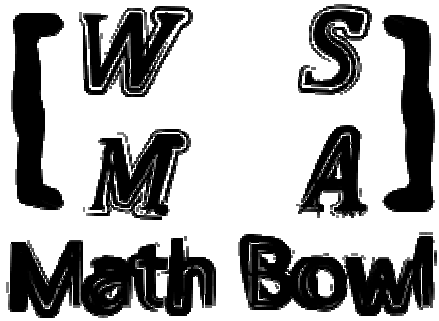
Convert $\sqrt{524_6}$ into a base 3 number.

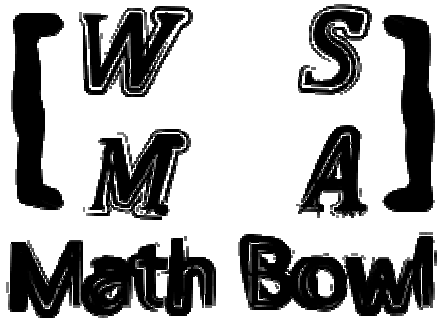What is the probability of getting a sum of 5 when rolling three fair standard six-sided dice?

Compute the following sum:
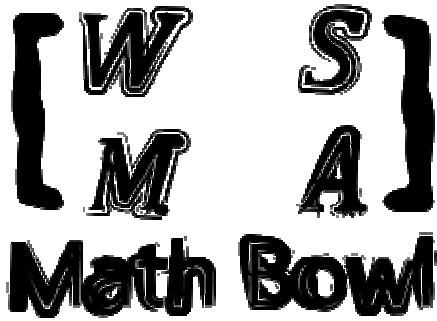
1+ 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25 + 28 + 31.

If *x* is a real number, find the minimum value of the following function: $f(x) = x^2 + 6x + 9$.

If *2x+4y= -6*, find the value of
$x^2+3xy+2y^2+4x+5y+3$.

There are 53 high school students taking at least one of Advanced Placement chemistry, calculus, or physics. 28 students take AP chemistry, 24 students take AP calculus, and 26 students take AP physics. 6 students take both AP chemistry and AP calculus, 9 students take AP calculus and AP physics, and 10 students take AP chemistry and AP physics. How many students take all three classes?
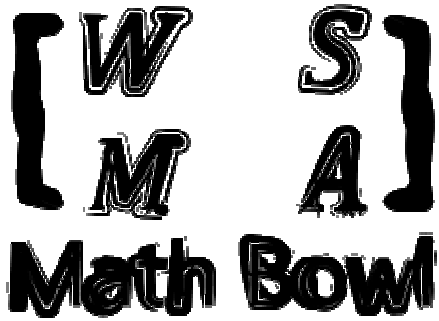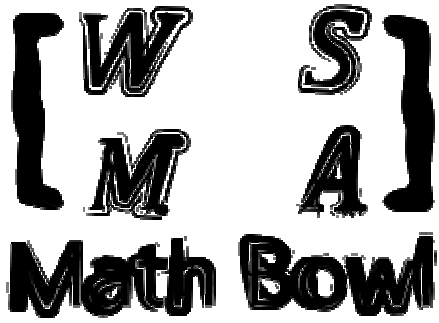
Find the determinant of the following matrix:

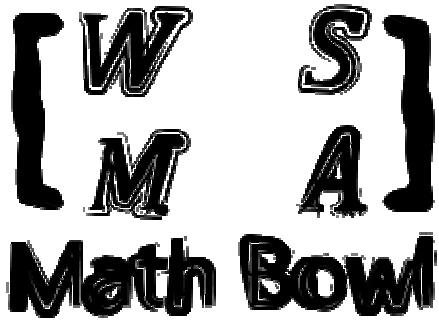$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$
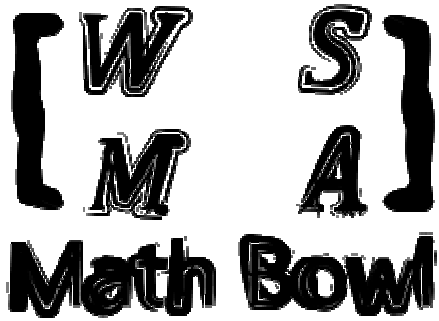
x is a positive integer less than 400 that has a remainder of 1 when divided by 2, 3, 4, 5, and 6. If *x* is divisible by 7, find *x*.
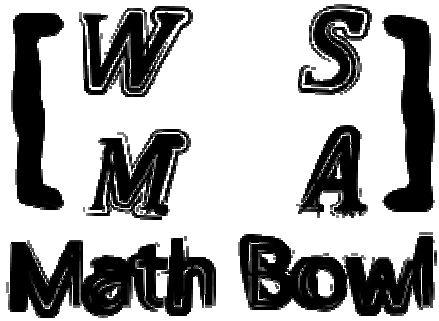
Evaluate the following infinite series:
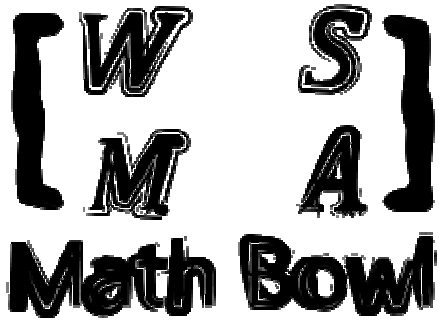
$$4 - \frac{8}{3} + \frac{16}{9} - \frac{32}{27} + \cdots.$$

Find the surface area of a hemisphere with radius 50.

Problem 15

How many arrangements of the letters in the word "MATHBOWL" are possible?

How many real values of *x* satisfy the equation
$\sqrt{x + 3} = x + k$ if $k < 3$?