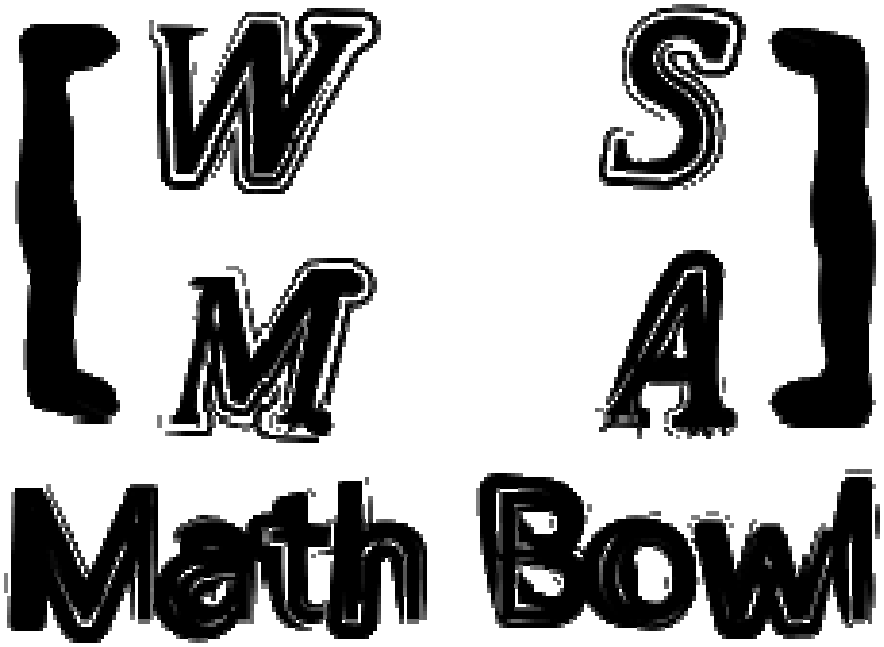

Elimination Tournament
Final Round

3nd Annual WSMA Math Bowl

March 2, 2013

Problem 1

In triangle ABC, D is the midpoint of BC. A median line is drawn from vertex *A* in triangle *ABC*. If *AB* = 8, *AC* = 6, *BD* = 5, and *DC* = 5, find the value of *AD*.

Problem 2

Evaluate: $2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$

Problem 3

On an analog clock, what is the first time after 4 o'clock at which the minute hand and hour hand coincide? Round your answer to the nearest minute.

Problem 4

Find the area of a triangle with side lengths 7, 8, and 9.

Problem 5

The four digit number *abba* is a multiple of 7. Find the maximum value of $a + b$.

Problem 6

If $\frac{a+b}{2} = \frac{b+c}{4} = \frac{c+a}{5}$, find the ratio a:b:c.

Problem 7

If $1.23^x = 100$ and $0.00123^y = 100$, find the value of $\frac{1}{x} - \frac{1}{y}$.

Problem 8

Steven and Andrew are creating math problems for WSMA. Steven works for 2 days and rests the next day. Andrew works for 5 days and rests the next 2 days. If they start on the same day, after how many days will they have rested simultaneously for the 5^{th} time?

Problem 9

The roots of the equation $x^3 - 3x^2 - 6x + k = 0$ form an arithmetic sequence. Find the value of k.

Problem 10

Steven and Andrew plan to meet at WSMA to attend the Math Bowl. Instead of determining a meeting time, Steven and Andrew each plan to arrive at some time between 12pm and 2pm, inclusive. After they arrive, Steven and Andrew will wait for 15 and 30 minutes respectively before leaving if the other does not show up. What is the probability that Steven and Andrew will meet and attend the Math Bowl together?

Problem 11

The n^{th} term of the Fibonacci sequence can be found using the equation

$$F_{\text{n}} = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n}.$$

Find the 15^{th} term of the Fibonacci sequence.

Problem 12

Find the sum of all four-digit palindromes.

Problem 13

Two cups are placed on a table. Cup *A* contains 100g of saltwater with an *x*% concentration while cup *B* contains 200g of saltwater with 12.5% concentration. When the two solutions inside the two cups are mixed, the solution will have a concentration of 30%. Find the value of *x*.

Problem 14

A leaky container can be filled using one or two hoses. It takes 3 hours to fill the container with hose *A* and 6 hours to fill the container with hose *B*. However, a hole in the container will empty the full container in 4 hours. How many hours will it take to fill the container using both hoses *A* and *B*?

Problem 15

Compute the following infinite sum: 0.4 + 0.01 + 0.006 + 0.0006 + 0.00006 + 0.000006 +….

Problem 16

For any integer n, let $f(n) = n - 2$, and let $f(x) \diamond f(y) = f(xy)$. If x and y are integers, find the value of $1 \diamond 2$.

Problem 17

A cylindrical muffin is 2 inches tall and 4 inches in diameter and costs $2.00. If its height decreases in half and the diameter increases by 1 inch, but its price remains the same, how much extra revenue is the greedy cafeteria making, assuming that a muffin costs the same the same amount to make per unit volume? Express your answer to the nearest cent.

Problem 18

Arthi's dinner was a slice of BBQ chicken pizza and three garlic knots, and cost $4.75. Jin had a slice of Grandma's pizza and five garlic knots for $3.50, and Sophia had a slice of Grandma's pizza and two garlic knots for $2.75. How much do two slices of BBQ chicken pizza, one slice of Grandma's pizza, and seven garlic knots cost?

Problem 19

How many more ways are there to try three flavors out of 50 at Ralph's Italian Ices than at Baskin Robbins, where there are (theoretically) 31 flavors?

Problem 20

Steven is making honey milk tea. He first makes a 12 oz. cup of tea with no milk, then mixes 8 oz. of tea with 10 oz. of milk. How many times more honey would Hansen get if he added the honey in the 18 oz. cup than if he had added it in the 12 oz. cup? Assume honey takes up a negligible volume.

Problem 21

Susan has a farm with turtles and penguins. If there are 90 legs and 28 heads, how many turtles are there?

Problem 22

Rose has 10 identical York candies that she wants to give to share among her 4 friends. In how many ways can she do this such that each friend receives at least 1 candy?