WSMA Math Bowl - March 2, 2013

Bonus Room Proctors' Answer Sheet

1	The sum of three positive integers is 1111. What is the greatest possible value for the greatest common factor of the three integers?	101
2	Find the value of $p + q$ if p, q, and $p^q + q^p$ are all prime numbers.	5
3	Find the number of rectangles that can be formed from the diagram below:	18
4	What is the smallest positive integer x for which $x^2 + x + 41$ is a composite number?	40
5	For $k = 1, 2, 3,$ or 4, let a_k be an integer that satisfies $0 \leq a_k \leq k$. If $\frac{67}{24} = \frac{a_1}{24} + \frac{a_2}{12} + \frac{a_3}{4} + a_4$, find the value of $a_1 + a_2 + a_3 + a_4$.	6
6	What is the sum of the maximum number of regions and points of intersection formed if four lines lie on a plane?	17
7	How many times will the function $f(x) = 2x^2 - x + 19$ intersect the x-axis?	0
8	Water is flowing into a cylindrical can at a rate of 3 cubic centimeters per second. If the can has a base with radius of 5 cm, at what rate is the height of the water in the can increasing? Express your answer in terms of centimeters per second.	$\frac{3}{25\pi}$
9	Susan has 4 pigs, and two of them are twins. The youngest pig is 1 year old while the oldest one is 2 years younger than 3 times the age of the twin pigs. The sum of the ages of the pigs is 29 years. Find the age, in years, of the twin pigs.	6
10	Daniel has 3 different Taylor Swift songs, 1 Straight No Chaser song, and 4 different Hans Zimmer songs on his playlist. What is the probability that, when he shuffles this playlist, the Taylor Swift songs will end up together?	3/28
11	Square ABCD in the coordinate plane has three of its vertices at points A(1, 3), B(5, 7) and C(9, 3). Find the coordinates of vertex D.	(5,-1)
12	If $f(x + y + z) = f(x) + f(y) + f(z) + 1$, $f(1) = \pi$, and $f(\sqrt{5} - 2) = 3$, find the value of $f(\sqrt{5})$.	$2\pi+4$
13	There are 10 Guardian Angels and 12 Demons under the temple of WSMA. Two beings, chosen among them, are selected to be sent to Evan. When Evan asks truth-telling WSMA members if at least one Demon was selected, the members reply yes. What is the probability that both selected beings are Demons?	11/31

14	3 Siege Tanks attack 12 Zerglings, killing 2 of them and damaging the rest so that the remaining 10 Zerglings have unequal amounts of health remaining. How many ways can Arjun divide the remaining Zerglings into 3 groups of 3, 3 and 4 Zerglings, respectively?	4200
15	If $1 < x < 3$, simplify $\sqrt{1-2x+x^2} + \sqrt[4]{(x^2-6x+9)^2}$.	2
16	Solve the following equation for x: $987x^2 - 251x - 736 = 0$.	1, -736/987
17	Let *A* be the number of sides of a regular polygon with exterior angles measuring 36 degrees and let *B* be the sum of the infinite geometric series with initial value $\frac{1}{2}$ and common ratio 1/2. Find *A*+*B*	11
18	Evaluate the following: $1^3 + 2^3 + 3^3 + \cdots + 15^3$	14400
19	The line $y = 2x + b$ passes through the points $(-1, 3)$ and $(a, -3)$. Find the value of $\frac{b}{a}$	-5/4
20	At 7 p.m., Brian realizes he needs to prepare a presentation for 8 a.m. the next day. His productivity is currently 90% and it decreases to 75% after 10 p.m. and it decreases again to 50% after 3 a.m. If he starts working right away and finishes at 8 a.m., how much time would he have saved by not procrastinating and working at 100% productivity? Express your answer as a decimal.	4.05 hours
21	In triangle ABC, D is the midpoint of BC. A median line is drawn from vertex *A* in triangle *ABC*. If *AB* = 8, *AC* = 6, *BD* = 5, and *DC* = 5, find the value of *AD*.	5
22	Evaluate: $2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \cdots}}}$	4
23	On an analog clock, what is the first time after 4 o'clock at which the minute hand and hour hand coincide? Round your answer to the nearest minute.	4:22
24	Find the area of a triangle with side lengths 7, 8, and 9.	$12\sqrt{5}$
25	The four digit number *abba* is a multiple of 7. Find the maximum value of *a* + *b*.	16
26	If $\frac{a+b}{2} = \frac{b+c}{4} = \frac{c+a}{5}$, find the ratio *a*:*b*:*c*.	3:1:7
27	If $1.23^x = 100$ and $0.00123^y = 100$, find the value of $\frac{1}{x} - \frac{1}{y}$.	3/2
28	Steven and Andrew are creating math problems for WSMA. Steven works for 2 days and rests the next day. Andrew works for 5 days and rests the next 2 days. If they start on the same day, after how many days will they have rested simultaneously for the 5th time?	48 years
29	The roots of the equation $x^3 - 3x^2 - 6x + k = 0$ form an arithmetic sequence. Find the value of *k*.	8

30	Steven and Andrew plan to meet at WSMA to attend the Math Bowl. Instead of determining a meeting time, Steven and Andrew each plan to arrive at some time between 12pm and 2pm, inclusive. After they arrive, Steven and Andrew will wait for 15 and 30 minutes respectively before leaving if the other does not show up. What is the probability that Steven and Andrew will meet and attend the Math Bowl together?	43/128
31	The n^{th} term of the Fibonacci sequence can be found using the equation $$F_{\mathrm{n}} = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{n}.$$ Find the 15^{th} term of the Fibonacci sequence.	610
32	Find the sum of all four-digit palindromes.	495000
33	Two cups are placed on a table. Cup *A* contains 100g of saltwater with an *x*% concentration while cup *B* contains 200g of saltwater with 12.5% concentration. When the two solutions inside the two cups are mixed, the solution will have a concentration of 30%. Find the value of *x*.	65
34	A leaky container can be filled using one or two hoses. It takes 3 hours to fill the container with hose *A* and 6 hours to fill the container with hose *B*. However, a hole in the container will empty the full container in 4 hours. How many hours will it take to fill the container using both hoses *A* and *B*?	4
35	Compute the following infinite sum: 0.4 + 0.01 + 0.006 + 0.0006 + 0.00006 + 0.000006 +….	5/12
36	For any integer *n*, let $f(n) = n - 2$, and let $f(x) \blacklozenge f(y) = f(xy)$. If *x* and *y* are integers, find the value of $1 \blacklozenge 2$.	10
37	A cylindrical muffin is 2 inches tall and 4 inches in diameter and costs $2.00. If its height decreases in half and the diameter increases by 1 inch, but its price remains the same, how much extra revenue is the greedy cafeteria making, assuming that a muffin costs the same the same amount to make per unit volume? Express your answer to the nearest cent.	44
38	Arthi's dinner was a slice of BBQ chicken pizza and three garlic knots, and cost $4.75. Jin had a slice of Grandma's pizza and five garlic knots for $3.50, and Sophia had a slice of Grandma's pizza and two garlic knots for $2.75. How much do two slices of BBQ chicken pizza, one slice of Grandma's pizza, and seven garlic knots cost?	$12
39	How many more ways are there to try three flavors out of 50 at Ralph's Italian Ices than at Baskin Robbins, where there are (theoretically) 31 flavors?	15105
30	Steven is making honey milk tea. He first makes a 12 oz. cup of tea with no milk, then mixes 8 oz. of tea with 10 oz. of milk. How many times more honey would Hansen get if he added the honey in the 18 oz. cup than if he had added it in the 12 oz. cup? Assume honey takes up a negligible volume.	$\frac{3}{2}$
41	Susan has a farm with turtles and penguins. If there are 90 legs and 28 heads, how many turtles are there?	17

42	Rose has 10 identical York candies that she wants to give to share among her 4 friends. In how many ways can she do this such that each friend receives at least 1 candy?	84