Elimination Round 3

2nd Annual WSMA Math Bowl
April 28, 2012

Problem 1

Steven forgot to write down an important phone number. However, he remembers that it started with 206 and that the next 7 digits are formed with the numbers 1, 4, 5, 7, 8, 9 with one appearing twice. If he guesses a phone number, what is the probability that he gets the correct number?

Problem 2

Sean and Ben are running around a track with a perimeter of 500 meters. If they start simultaneously from the start line in the same direction, Ben will pass Sean in 4 minutes. If they run in different directions, they will meet in 2 minutes. What is Ben's speed in m/s?

Problem 3

In triangle ABC, points B and C are fixed, BC is length 8, and point A is any point such that $AB + AC = 12$. What is the maximum possible integer area of triangle ABC?

Problem 4

Hansen has 10 identical York candies that he wants to share among his 4 friends. In how many ways can he do this such that each friend receives at least 1 candy?

Problem 5

The sum of seven consecutive even integers is 994. How many of these integers have 5 or more factors?

Problem 6

If a and b are consecutive integers where $a < b$ and c is the average of a and b, what is the positive difference between $(b^2 - c^2)$ and $(c^2 - a^2)$?

Problem 7

Andrew wants to distribute 12 identical balls into 4 boxes so that each box contains at least 2 balls per box. Samir, however, wants to place the 9 balls into 3 boxes so that there are at least 1 ball per box. What is the sum of the possible ways Andrew and Samir would each distribute the balls?

Problem 8

Let *S* be a list of positive integers that are not necessary distinct in which the number 38 appears. The arithmetic mean of the numbers in _ is 29. However, if 38 is removed, the arithmetic mean of the numbers is 28. What is the sum of the digits of the largest number that can appear in *S*?

Problem 9

How many pairs of integers (x, y) satisfy the following equation: $|x - 1| + |y + 2| = 2$?

Problem 10

How many solutions does the equation $a+b+c+d+e=21$ have where a, b, c, d, and e are all whole numbers?

Problem 11

If $f(x) = x^{\frac{1}{2}} + x^{\frac{3}{2}} + x^{\frac{5}{2}} + \cdots$, what is $f\left(\frac{1}{4}\right)$?

Problem 12

A scale weight that weighs 13g broke into 3 parts after accidently dropping it on the floor. Surprisingly, the 3 new weights were found to weigh integer values and also capable of measuring any kind of integer weight equal or less than 13g. How much does each scale weight weigh?