Problem Solving Method 1:

<u>Test the first few values of "n" and look for patterns</u>

Example: Find the remainder when 7^{2008} is divided by 19.

- Test: What are the remainders of 7^n when divided by 19?

- When n = 1, the value becomes 7 and the remainder is ***7***
- When n = 2, the value becomes 49 and the remainder is ***11***
- When n = 3, the value becomes 343 and the remainder is ***1***
- When n = 4, the value becomes 2401 and the remainder is ***7***
- When n = 5, the value becomes 16807 and the remainder is ***11***

At this point, we notice a recurring pattern of 7, 11, and 1 as the remainders. This cycle repeats itself for every three values of "n".

Assuming this pattern holds, we find the remainder when 7^{2008} is divided by 19 by taking the remainder of 2008/3, which is 1.

Therefore, 7^{2008} has the same remainder as 7^1 and the answer is $\boxed{7}$, the remainder we found when n = 1. Thus, C is the correct answer.

Problem Solving Method 2:

Use only the necessary information to avoid complex calculations

It is difficult to calculate larger powers and take remainders of larger numbers without using a calculator. Fortunately, there is a shorter method.

It is possible to calculate remainders of powers without evaluating the powers themselves.

Why is this? Each time a number is divided, it is split into a **quotient** (a number divisible by the divisor, ex. 19) and a **remainder** (a positive integer smaller than 19).

The remainder is the only necessary information. The quotient is irrelevant, because **it is already divisible by 19 and multiplying it by 7 won't change its divisibility**.

Thus, all we need to do in order to find the remainder of 7^{n+1} is to multiply the remainder of 7^n by seven, and divide by 19 if necessary.

Our problem now is much simpler, and looks like:

- Test: What are the remainders of 7^n when divided by 19?

- When n = 1, the value becomes 7x1=7 and the remainder is ***7***
- When n = 2, the value becomes 7x7=49 and the remainder is ***11***
- When n = 3, the value becomes 7x11=77 and the remainder is ***1***
- When n = 4, the value becomes 7x1=7 and the remainder is ***7***
- When n = 5, the value becomes 7x7=49 and the remainder is ***11***

Using this method, the reason that the values repeat and follow a cycle of three becomes much clearer.

