Algebra – Two Variables

Last week we studied how to "solve for x" in equations with one unknown. This lesson begins to "solve for x and y" in problems of two equations and two unknowns.

System of Equations

Two or more equations that must all be true at the same time are called a **system of equations**. The values of the variables that make both equations true at the same time are the **solution of a system**.

$y = 4x$	
$x + y = 90$	$x = 18, y = 72$
A system of equations.	*The solution of the system.*

Many times the system of equations involves two equations and two unknowns. That's what we'll study in Math Club.

There are several methods to solve a system of equations. Some of the methods may seem familiar, and some may be new. They are all effective with two equations and two unknowns. We will focus on "adding equations" and "variable substitution":

1. Guess and check
2. Solve a simpler problem
3. Draw a picture
4. Draw a graph
5. Adding equations
6. Variable substitution

Adding Equations

Algebra permits you to modify any equation, as long as you do the same thing to both sides, right? Believe it or not, this allows you to add two equations together.

The reasoning for adding two equations together goes like this. An equation is a statement of equality. The stuff on the left side *equals* the stuff on the right. The two sides are interchangeable. So when you add one equation to another, you are really adding the same amount (whatever unknown amount it is) to both sides.

Remember our analogy to a balance? Since both equations were in balance to begin with, the sum is still in balance. Although you may not know how many "pounds" you're adding to both sides of the balance, you are adding the same number to both sides. It does not upset the balance; both sides remain equal.

The goal for adding equations is to eliminate one of the variables. So this method works when one equation has a variable that has the opposite value from the other equation. For example, if one equation contains "$-7x$" and the other contains "$+7x$" then adding the equations causes variable x to vanish, leaving you with one equation and one variable.

After you solve the remaining one equation for the one unknown, how do you solve for the other unknown? You can substitute the value into either one of the original two equations, and solve for the last unknown.

Example:

$A + B = 50$
$A - B = 22$
What are the values of A and B?

Solution:

	Add both equations together:	$\begin{array}{r} A + B = 50 \\ + \underline{\ A - B = 22} \\ 2A + B - B = 50 + 22 \end{array}$
	Combine similar terms:	$2A = 72$
	Solve for A:	$A = 36$
	Substitute back to find B:	$36 + B = 50$ $B = 50 - 36 = 18$
Check:	Check the result with both:	$36 + 18 = 50?$ *Yes!* $36 - 18 = 22?$ *Yes!*

This worked great on a system of equations that (ever so conveniently) had terms that canceled out after addition. But what if an unknown *doesn't* cancel when you add the equations together?

The trick is to *multiply by a constant*. Look at your two equations before you add them. Choose one, and figure out what constant to multiply by. Select a constant (it might be negative) such that one unknown will cancel out when you add the equations. Multiply both sides of your chosen equation by the constant, and then add the two equations together.

Example:

$x + 4y = 16$
$2x - y = 5$
What are the values of x and y?

Solution:

	Look at the equations. What constant should we use? Let's multiply the second equation by 4:	$4(2x - y) = 4\,(5)$ $8x - 4y = 20$
	Add both equations together:	$x + 4y = 16$ $+\ 8x - 4y = 20$ $9x + 4y - 4y = 16 + 20$
	Combine similar terms:	$9x = 36$
	Solve for x:	$x = 36/9 = 4$
	Substitute $x = 4$ to find y:	$4 + 4y = 16$ $4y = 12$ $y = 12/4 = 3$
Check:	Check the result with both:	$4 + 4(3) = 16$? *Yes!* $2(4) - 3 = 5$? *Yes!*

Variable Substitution

You can solve a system of equations with a method called *substitution*. Here's how to use it.

You have two equations, so "solve" one equation by writing A in terms of B. That is, use the rules of algebra to rearrange the equation to get "A =" on one side. Then substitute this expression into the other equation wherever variable A appears. This results in one equation and one unknown.

Example: $y = 4x$
$x + y = 90$
What are the values of x and y?

Solution: The first equation $y = 4x$ tells you that "$4x$" is another name for y.

Substitute $4x$ for y:
$x + y = 90$
$x + 4x = 90$
$5x = 90$

Divide both sides by 5:
$5x/5 = 90/5$
$x = 18$

To find y, substitute 18 for x in either of the original equations:
$y = 4x$
$y = 4(18)$
$y = 72$

Check: Substitute the values of x and y into the other equation:
$x + y = 90$?
$18 + 72 = 90$? *Yes!*

Algebra Problem

Here's a problem that can be solved by adding equations:

> Tom has some quarters and nickels. He has 33 coins, and their total value is \$5.05. How many quarters does he have?

Let Q = the number of quarters, and N = the number of nickels. How can we get two equations out of this?

- Notice the first part said he has 33 coins. That means the number of coins must total 33, which can be written as: Q + N = 33.

- Notice the second part tells their total value. That means the number of quarters multiplied by 25¢, plus the number of nickels multiplied by 5¢ must add up to the total value. We can write this as 25Q + 5N = 505.

Now we have two equations and two unknowns, which we can solve.

25Q + 5N = 505
Q + N = 33
Find the values of Q and N?

Look at the equations.
What constant should we use?
Let's multiply the second equation by -5:

-5(Q + N) = -5(33)
–5Q –5N = -165

Add both equations together:

$$\begin{array}{rrcr} 25Q & +5N & = & 505 \\ -5Q & -5N & = & -165 \\ \hline 20Q & & = & 340 \end{array}$$

Solve for Q:

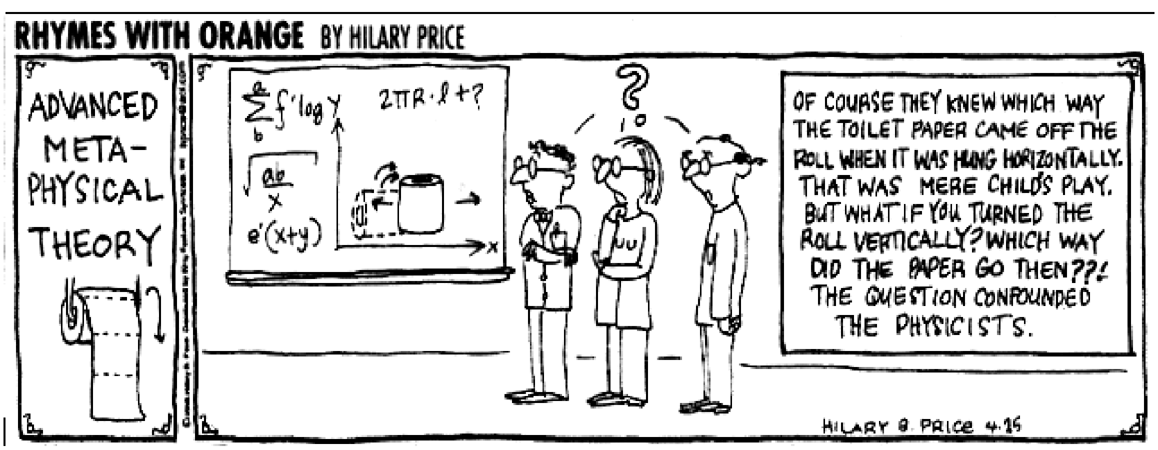
20Q/20 = 340/20
Q = 17 quarters

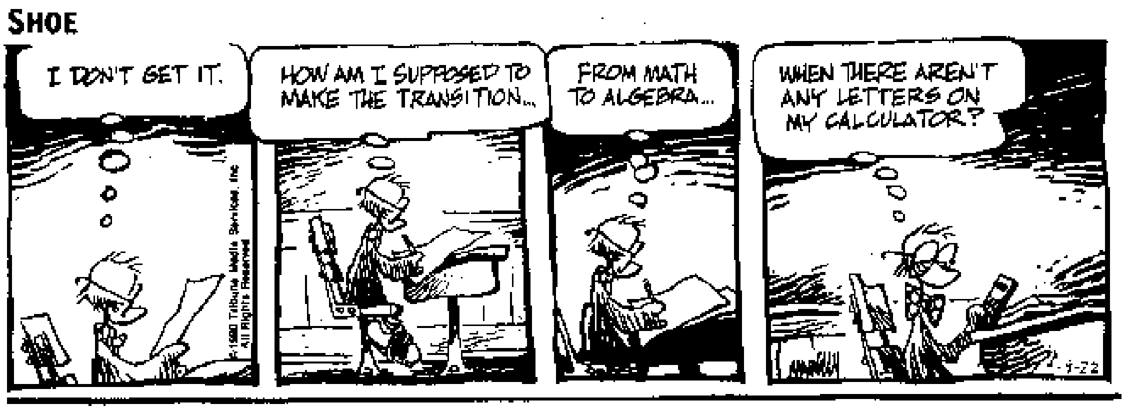
Substitute Q = 17 to solve for N:

17 + N = 33
N = 33 – 17 = 16 nickels

Check the result with both:

25(17) + 5(16) = 505? *Yes!*
17 + 16 = 33? *Yes!*





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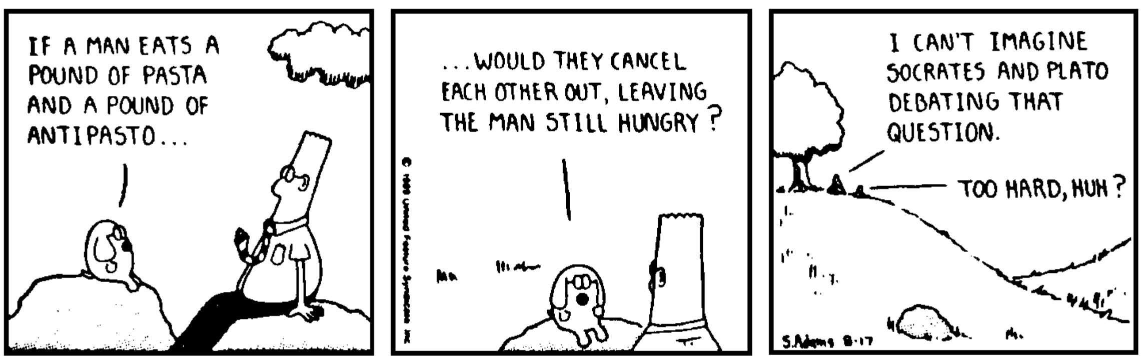

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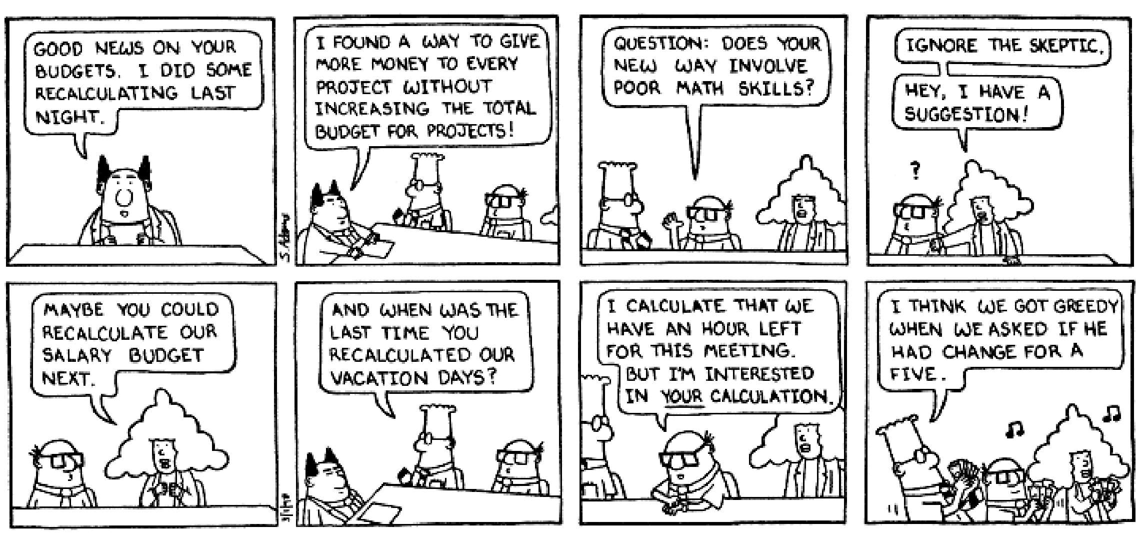

1) Solve for x. These problems take two or more steps to solve.
Hint: Review your notes from last week for a reminder of the process.

a) $8x - 5 = 19$

Example: $8x - 5 + 5 = 19 + 5$
$8x = 24$
$x = 24/8 = 3$

Check: $8(3) - 5 = 19$ √

b) $3 + 6x = 21$

c) $8x + 4 = 10x$

d) $12(x - 4) = 24$

e) $\frac{144}{x} = 6$

f) $6x = 3x + 21$

g) $\frac{(5x+4)}{8} = 3$

h) $12x + 5 = 2x$

2) These students are trying to solve systems of equations, but they're not in Math Club! Check their work and circle "correct" or "incorrect".

a) $a = 3b$ Andrew's solution:
$4a + 2b = 28$ $a = 10,\ b = 2$ Correct? Incorrect?

b) $12r - 8t = -52$ Zena's solution:
$5t = r$ $t = -1,\ r = -5$ Correct? Incorrect?

c) $y = 13 - 2x$ Kareem Abdul Jabbar's solution:
$y = 5x + 41$ $y = 5,\ x = -4$ Correct? Incorrect?

3) Multiply both sides of each equation by the constant.
Simplify where possible. Always reduce fractions.

a) $x + 3y = 13$ *Example:* $6\,(x + 3y) = 6\,(13)$
Multiply by 6. $6x + 18y = 78$

b) $2x + 3y = 24$
Multiply by 10.

c) $6x + 24y = 60$
Multiply by 1/6.

d) $10x - 7y = 1$
Multiply by -5.

e) $22x - 33y = 121$
Multiply by 1/11.

f) $-220x + 340y = 190$
Multiply by 0.1

4) Use the "adding equations" method to solve for <u>both</u> *x* and *y*.

a) $x + y = 16$
$x - y = 2$

c) $9x - y = 3$
$2x + y = 19$

b) $5x + 2y = 15$
$3x - 2y = 1$

d) $-2x + 5y = 12$
$2x - y = 4$

5) Use the "adding equations" method to solve for both *x* and *y*.
Hint: You first need to multiply one of the equations by a constant, to make one of the unknowns cancel when you add.

a) $3x + 2y = 110$
$x - y = 20$

b) $2x + 5y = 29$
$-x + 4y = 5$

c) $3x + 5y = 85$
$3x + 2y = 52$

d) $-11x + 4y = 69$
$4x - y = 34$

6) Mental Math: do these in your head, and write down the answers. When you're all done, *check your answer* with pencil and paper, or with a calculator

a) If $6x = 72$, what is x?

b) Take 30% of 50 and add 5.

c) Take 1/3 of 270 and add 15.

d) What is your name?

e) What is the average of 10 and 34?

f) What is the average of 17, 20 and 23?

g) What number is the result of anything raised to the power of zero?
(*Hint:* The result is *not* zero!)

h) If $x = 5$, what is $4x^2 - 10$?

i) What is 5/8 plus 5/8, expressed as a mixed fraction?

Did you check my work?

