Order of Operations

Look at "$4+5\times 2$". Should you add 4 plus 5 and then multiply by 2, or multiply first and then add? How do you know which to do first?

Natural Order

There is a "natural" order of operations to mathematics:

↑ *PEMDAS* ↑

1. If there's any ***p***arentheses ( ) then do them first.
2. Then do ***e***xponents x^n and powers 10^3 and radicals $\sqrt{}$.
3. Then do ***m***ultiplication $\times$ and ***d***ivision $\div$.
4. And then do any ***a***ddition + and ***s***ubtraction - from left to right.

It may help to remember them as ***P.E.M.D.A.S.*** using their first letters.

Thus in the example $4+5\times 2$, we would multiply first.

Example: 8 - 4 - 1 = ?

Answer: Since we have more than one subtraction, we do the *left* one first:
(8 - 4) - 1 = 3

Parentheses

What if we wanted to tell someone to do the subtraction in a different order? Use *parentheses* to change the natural order. Expressions in parentheses are always done first. Note: Singular is *parenthesis*, and plural is *parentheses*.

Example: 8 - (4 - 3) = ?

Answer: The parentheses tell us to do "*four minus one"* first:
8 - (4 - 3) = 7

What about nested parentheses? Evaluate the *innermost* parentheses first.

Example: $(8-(4+2))^2 = ?$

Answer: $(8-6))^2 = (2)^2 = 4$

Exponents

Suppose you have an expression with both subtraction and exponents. You should do the exponent first (unless commanded otherwise by parentheses).

Example: $25-2^4=?$

Answer: Since there is no parentheses ( ) we do "*two to the fourth power*" first:
$25-16=9$

Calculator Warning!

See <u>your</u> calculator's manual to read about its order of operations. The cheapest ones simply do everything left-to-right as you enter the operations. The better ones save the results of multiplication and then add (or subtract) the products together. Try this on your calculator:

$$2\times3+4\times5=?$$

A very basic calculator solves it like this, as you enter it from left-to-right:

$2\times3+4\times5$
$6+4\times5$
10×5
<u>50</u>

A better calculator will follow the natural order of operations, and give a different answer even though you pressed the same keys in the same order:

$2\times3+4\times5$
$6+4\times5$
$6+20$
<u>26</u>

So be sure to understand how <u>your</u> own calculator handles its order of operations!

By the way, most calculators that follow the natural order (multiplication first) <u>also</u> have parentheses keys so you can change the order, if needed. If you see parentheses on a calculator, that's a good clue that it was designed to handle the natural order of operations. If in doubt, use the parentheses keys "(" and ")" to ensure it follows the order you want.

What about the fancy HP calculators that don't have parentheses or equals keys? They are known as Reverse Polish Notation (RPN) data entry. It's up to <u>you</u> to control the order of operations! HP calculators keep their numbers in a stack; you arrange things such

that the two numbers you want to multiply is at the end of the stack. Then you push × to multiply, and it consumes two numbers and puts the result back on the stack.

More Examples

If **a = 4** and **b = 3** compute the result:

Example 1: $a+6-b=?$
Answer: Add and subtract from left to right: $4+6-3=7$

Example 2: $a/2\times b=?$
Answer: Multiply and divide from left to right: $4/2\times 3=6$

Example 3: $b-a/2=?$
Answer: The division comes first, then the subtraction: $3-4/2=1$

Handling Fractions

One last complication: What about the expression $\frac{6+4}{5-3}$?

To figure this out, we must tell you there are always implied parentheses around the numerator and denominator of a fraction. So first do addition and subtraction, and then do the division: $\frac{(6+4)}{(5-3)}=\frac{10}{(5-3)}=\frac{10}{2}=5$

What about cascading fractions such as $\frac{1}{\frac{2}{3}}$? This can be either $\frac{1}{(2/3)}$ or $\frac{(1/2)}{3}$ which give different answers. The writer must be clear when putting it on paper! This is another great reason to love parentheses.

Example: $\frac{1}{2+\frac{3}{4+5}}=?$

Answer: Use the implied parentheses around each fraction, then start with the innermost and work your way out. $\frac{1}{\left(2+\frac{3}{(4+5)}\right)}=\frac{1}{\left(2+\frac{1}{3}\right)}=\frac{1}{\left(\frac{7}{3}\right)}=\frac{3}{7}$

Finding the Median

Two statistics that help to understand the meaning of numbers are the *mean* and the *median*. We studied the mean (average) in Lesson 12, but here is a quick refresher.

LINCOLN FOOTBALL TEAM	
Game	**Points Scored**
1	36
2	53
3	42
4	58
5	45
6	40
7	48

Recall that the **mean** is the average score:

$$\text{Avg} = \frac{V_1 + V_2 + \ldots + V_n}{n}$$

$$\text{Avg} = \frac{36 + 53 + 42 + 58 + 45 + 40 + 48}{7}$$

$$\text{Average Score} = \frac{322}{7} = 46$$

To find the **median**, we sort the scores and list them from lowest to highest. The median is the middle score.

36, 40, 42, 45, 48, 53, 58

Your choice depends on whether there are an *even* or *odd* number of values:

- For an ***odd*** number of scores, the median is the middle score.
- For an ***even*** number of scores, the median is the average of the two middle scores.

Are the mean and median really the same? Nope. Although they give similar answers, they are sometimes very different. The *median* is often used in the real estate business – it tells you what price of house "most" people buy. If they used the *mean* housing price, it would be unrealistically large because just a few extremely expensive houses would raise the average quite a bit. (Just think how much Bill Gates' house would affect an average!)

The Duplex

Vocabulary

- *Arithmetic mean* – when one number is cruel to another. Example: Why was ten afraid of seven? Because seven eight nine!

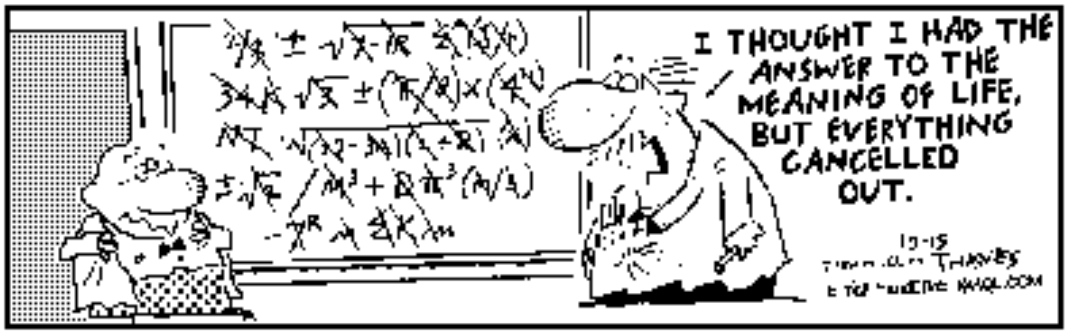
- *Arithmetic progression* – A sequence of number in which the difference of two consecutive numbers is the same. The difference of the two consecutive numbers is called the *common difference*. Example: 3, 6, 9, 12, 15, …

- *Geometric progression* – A sequence of number in which each succeeding term is obtained by multiplying the preceding term by the same number. That number is called the *ratio* or *common ratio* of the geometric progression.
  Example: 64, 32, 16, 8, 4, 2, 1, ½ , ¼, …

- *Independent events* – In probability, when the outcome of one event does not depend on the outcome of another event. For example, in two tosses of a coin, the outcome of the second toss does not depend on the outcome of the first toss. In each toss the probability of tails landing up is ½ and the probability of heads landing up is ½.

- *Repeating decimal* – A decimal number that just won't shut up. It has one or more digits repeating without end. It has one or more digits repeating without end. It has one or more digits repeating without end. It has one or more digits repeating without end. And so forth!

Fred Bassett, by Alex Graham



1) Find the result, using the natural order of operations.

a) $3\times 8-4=$ *Example: 24 - 4 = 20*

b) $7-1-8=$ _____

c) $36/4-5=$ _____

d) $3+3\times 3-3/3=$ _____

e) $(14-2)/(5+5)=$ _____

f) $10-(4+2)\times 2=$ _____

2) Find the result, using the natural order of operations and parentheses.
Hint: Look at lesson 6 for exponents of zero!

a) $5^2+3=$ *Example: 25 + 3 = 28*

b) $1-10^2=$ _____

c) $(2+4)^2-8=$ _____

d) $(4+12)^0+1=$ _____

e) $8^2+4^1/17^0=$ _____

f) $4^3\times 2^{-3}=$ _____

g) $\dfrac{2}{2+\dfrac{2}{(2+2)}}=$ _____

3) Find the mean and the median for each set of numbers.
Hint: For an even number of values, the median is halfway between the middle two values.

a) 20, 22, 30 *Example:* *mean = 24* *median = 22*

b) 12, 13, 14, 15 mean = _____ median = _____

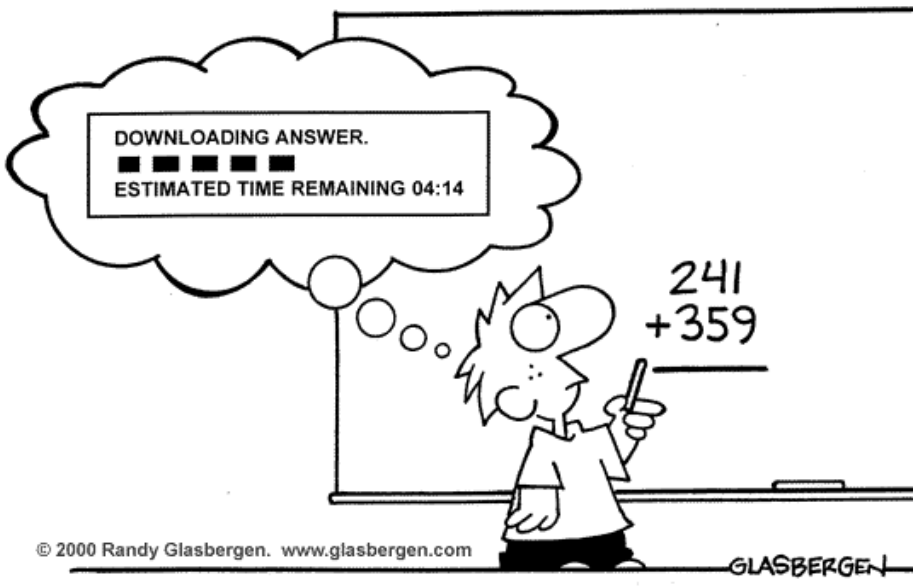
c) 16, 27, 19, 23 mean = _____ median = _____

d) $7.12, $7.86, $8.45, $12.81, $3.01
mean = _____ median = _____

e) 109, 130, 114, 106, 121, 116
mean = _____ median = _____

f) 13747, 11985, 15002 mean = _____ median = _____

g) If there are only two values, is their median always the same as their mean? Circle **yes** or **no**.



4) Mental Math: do these in your head, and write down the answers.
Leave all answers as reduced fractions, and in terms of radicals and pi.

a) What is 28×4?

b) What is $241 + 359$?

c) What is 8% of 50?

d) What is your name?

e) What is $\frac{1}{8}$ of 100?

f) Find 10% of 950, then subtract 30.

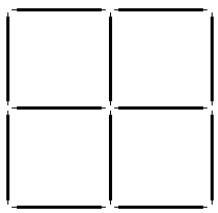
g) What is the GCF (greatest common factor) of 22 and 24?

h) How many miles will a car travel in 3 hours that averages 55 mph?

i) Work "drawkcab" to solve this problem:

The Backward Boy doesn't enjoy books very much. He always reads the endings first! He read 3 less pages in his cowboy book than he did in his science fiction book. The number of pages he read in his science fiction book was ½ the number of pages he read in his detective novel. He only read the last 20 pages of his detective novel. How many pages did the Backward Boy read in his cowboy book?

j) *Extra credit:* Remove two toothpicks from this figure, leaving two squares (not rectangles) of different sizes. Draw the remaining toothpick figure.



Did you czech your werk?