Do you play games? Do you like to plan ahead? Do you ever do something (such as choosing clothes in the morning) based on a guess about the future?

Many games include an element of luck. Any estimate about the future includes some amount of chance. How can mathematics help us understand random events?

Probability

We use "probability" to help us understand what *might* happen.

Probability only applies to cases where we don't know the outcome. If you already know the result then you can't make a prediction!

> *Definition:* The **probability** of a certain event taking place is the ratio of favorable outcomes to the number of possible outcomes.

You compute probability by counting the number of **favorable** outcomes and dividing by the number of **possible** outcomes.

Probability is always shown as a decimal or fraction in the range of 0 to 1. This fraction can also be written as a percent number between 0 and 100.

Always express your answer as a decimal or reduced fraction.

1. The result must be zero or positive. There is no such thing as a negative number of favorable or possible outcomes.

2. It must be less than or equal to one. The number of favorable outcomes cannot exceed the number of possible outcomes.

We often use the letter P for probability, and write "P(success)" to say "the probability of success". Similarly you can write "P(heads)" to say "the probability of heads".

How to Count the Possible Outcomes

When you count the possible outcomes for an experiment, you must include as much detail as possible. It is important to keep each outcome distinct from all the others.

- For multiple coin tosses, pretend you have multiple coins and that each coin is counted separately. It is helpful to number the coin tosses to identify them.

- For multiple rolls of a die, pretend you have multiple dice and that each one is unique and individual.

We count the coin tosses (or dice rolls) separate from the others to ensure we count all the possibilities. For example, if you are flipping two coins then there are only three possible outcomes, right? *Wrong*! There are two ways to get "one head and one tail" and they must be counted separately, like this: HH, HT, TH and TT. So, two coins have four possible outcomes.

Each additional coin you toss *doubles* the number of outcomes. For N coin tosses there are always 2^N possible outcomes.

Example: If you are rolling three dice (or one die three times, same thing) how many outcomes are possible?

Solution: Remember to treat each die individually! Think of them as *first* roll and *second* roll and *third* roll. Then think of the possibilities by considering each roll in turn:
1) There are six outcomes for the *first* roll.
2) For every outcome of the *first*, there are six outcomes for the *second* roll. This makes $6 \times 6 = 36$ possible outcomes.
3) For every outcome of the combined *first* and *second* roll, there are six more outcomes for the *third* roll. This makes $36 \times 6 = 216$ possible outcomes.

Examples with Coins

Example: Suppose you flip a coin once. What is the probability of getting heads?

Solution: There are two possible outcomes: heads or tails.
There is one favorable outcome: heads.
The probability of heads = P(heads) = $\frac{\text{number favorable}}{\text{number possible}} = \frac{1}{2}$

Example: Suppose you flip a coin three times. What is the probability of getting two heads and a tails, in any order?

Solution: List *all* the outcomes and count the *desired* outcomes.

Flip 1-2-3	Outcome
H H H	--
H H T	favorable
H T H	favorable
H T T	--
T H H	favorable
T H T	--
T T H	--
T T T	--

P(two heads and tails) = 3 / 8

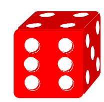
Example: Suppose you flip this coin *ten times* and it came up heads every time. What is the probability of getting heads on the next flip?

Solution: Coins have no memory. They don't change their behavior based on what already happened. There is still only one favorable outcome, and two possible outcomes, which continue to have equal probability.

$$P(\text{heads}) = \frac{favorable}{possible} = \frac{1}{2}$$

Examples with Dice

Example: What is the probability of rolling a 3 or higher on a 6-sided die? (Note, one is "die" and two is "dice".)



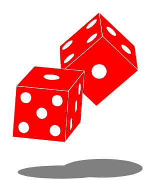
Solution: The favorable outcomes are 3, 4, 5, or 6. That is, four favorable outcomes in all. There are six possible outcomes.

$$P(\text{rolling a 3 or higher}) = \frac{4}{6} = \frac{2}{3}$$

Example: What is the probability of a sum of 7 when rolling two 6-sided dice?

Solution: Let's draw a table to count the number of favorable outcomes from two dice:

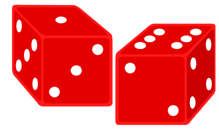
Sum	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12



The favorable outcomes are 1+6, 2+5, 3+4, 4+3, 5+2, or 6+1. That is, six favorable outcomes. With two dice there are 6 times 6, or 36 possible outcomes.

$$P(\text{rolling 7}) = \frac{6}{36} = \frac{1}{6}$$

Example: In the game of Risk® (a world conquest game), two players battle for territory by rolling dice and comparing their numbers. The highest number wins. To give a slight "home field advantage" to the defender, if both dice are equal then the defender wins.



What is the defender's probability of winning when both players roll just one die?

Solution: To find the number of favorable outcomes for the defender, we list all 36 possible outcomes, and count the ones where the defender wins. That is, when his dice shows the same number or higher than the attacker.

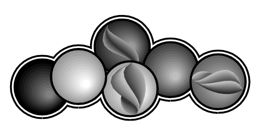
Attacker's Roll	**Defender's Roll** 1	2	3	4	5	6
1	win	win	win	win	win	win
2	lose	win	win	win	win	win
3	lose	lose	win	win	win	win
4	lose	lose	lose	win	win	win
5	lose	lose	lose	lose	win	win
6	lose	lose	lose	lose	lose	win

Counting the entries favorable to the defender:

$$\text{P(defender win)} = \frac{21}{36} = \frac{7}{12} = 58.\bar{3}\%$$

Examples of Selection

Example: George W. Bush is removing a marble at random from a box. The box contains 3 black, 4 red, and 5 striped marbles. What is the probability of taking a striped marble?



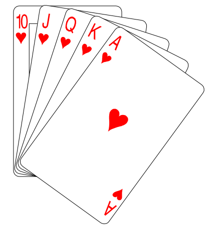
Solution: There are 5 striped marbles, and 12 marbles altogether.

$$\text{P(striped)} = \frac{\text{favorable outcomes}}{\text{possible outcomes}}$$

$$\text{P(striped)} = \frac{5}{12}$$

Example: Suppose you draw a card at random from a well-shuffled deck of 52 cards. What is the probability of drawing an ace or a heart on the first draw?

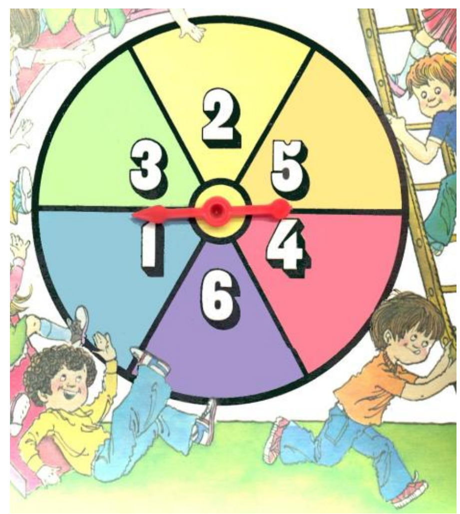
Solution: A standard deck has four suits of thirteen cards each. There are four favorable outcomes if you were to draw an ace, plus twelve other favorable outcomes (don't count the ace of hearts twice!) if you were to get a heart.



$$P(\text{ace or heart}) = \frac{4 \text{ aces} + 12 \text{ other hearts}}{52 \text{ cards}}$$

$$P(\text{ace or heart}) = \frac{16}{52} = \frac{4}{13}$$

Examples with Spinners



Some popular board games use a spinner to choose the next move. Spinners are easy for young players to use, and therefore often point to colors instead of numbers. Also, they cannot be swallowed as easily as dice!

Most spinners divide a 360-degree circle into equal-sized wedges. Some spinners will vary the odds of certain outcomes by adjusting the width of certain wedges. The spinner shown here is from the Chutes'N'Ladders game.

To compute probability from spinners, use the width of each wedge, as measured in degrees.

The spinner shown has six wedges of equal size. So divide 360 degrees by six wedges, and you find 60 degrees in each wedge. The probability of each is therefore

$$60/360 = 1/6 = 16.67\%$$

Example: What is the probability of spinning a 4 or a 5 on the Chutes'N'Ladders spinner shown above?

Solution: The favorable outcomes have 60 degrees for a 4, and 60 degrees for a 5. The total favorable outcomes are 120 degrees in a circle of 360 degrees.

$$P(4 \text{ or } 5) = \frac{120}{360} = \frac{1}{3} = 33\%$$

Vocabulary

- *Statistics* – Facts and figures gathered together for information on a particular subject, such as: “*the latest population statistics”*.
- *Probability* – The condition or odds of being likely to happen, or something likely to happen: “*Doughnuts with math club next week are a strong probability”.* See *odds.*
- *Odds* – The ratio of favorable outcomes to the possible outcomes. See *outcome*.
- *Outcome* – A final result; something that happened. See *likelihood*.
- *Likelihood* – The chance of something happening. See *chance*.
- *Chance* – The happening of things by luck or accident. See *random*.
- *Random* – Without plan, pattern or predicable result. See *luck*.
- *Luck* – The chance or random happening of good or bad outcomes. See *probability*.
- *Recursive* – See *recursive*.

Just For Fun…

If you say good-bye to Saturn and head further out, you find something else. What do you say hello to?

Using a mirror, turn this upside down and look at the reflection. All the words in the black panels can be read easily – but not those in the white panels. How and why do you think this happens?

BED	GREEN	DICE	RAIN
PEACE	EXCEDE	LION	DECK
BOX	CIRCLE	CODE	CHAIR
SWAN	CHICK	SMALL	CHOKED
DIXIE	DAISY	HOOD	CAT

Probably Homework

Find the probabilities of these single events. Write it as a reduced fraction.

1) You're rolling an 8-sided die. (Yes, there is such a thing!) What is the probability of rolling an even number?

   a) List the *favorable* outcomes: ______________________________

   b) How many *possible* outcomes: ______________________________

   c) P(any even number) = ______________________________

2) Now you're rolling *two* 8-sided dice. What is the probability of getting a sum of 7?

   a) List the favorable outcomes: ______________________________

   b) How many possible outcomes: ______________________________

   c) P(rolling a sum of 7) = ______________________________

3) On what type of dice are you more likely to roll a sum of 7: (*circle one)*
   (a) a pair of 8-sided dice, or (b) a pair of 6-sided dice?
   Hint: Look at the attached lesson for the 6-sided dice example.

4) Suppose you meet someone new. What is the probability your birthdays occur in the same month? *Hint:* You already know your own birthday.

   a) How many months of the year are "favorable outcomes"?
   ____

   b) How many possible months are in a year? _____________

   c) P(shared birthday month) = _____________

5) What is the probability of picking an Ace from a standard deck of 52 cards in one draw?

   a) How many favorable outcomes: _________________

   b) How many possible outcomes: _________________

   c) P(drawing an Ace) = _________________

6) Suppose each letter of our alphabet is written on a separate piece of paper and placed in a box. What is the probability that picking one piece of paper at random will have a vowel? (For this problem, don't count W or Y as vowels.)

7) Suppose you're playing a very simple two-person dice game called Odds'N'Evens. One person chooses to be "odd" and wins whenever the product of his two dice is odd. The other person chooses to be "even" and wins whenever the product of his two dice is even.

a) Finish this grid to indicate the odd and even products.

×	1	2	3	4	5	6
1	*odd*	*even*				
2						
3						
4						
5						
6						

b) What is the probability the "odd" person will win a toss of his two dice?

c) Is this a 'fair' game? Circle *yes* or *no*.

8) Recently, Laura and Steve were talking about their brothers and sisters. They noticed their families are reversed – Laura has three brothers and Steve has three sisters. Steve stated the odds were that most families with four children would have two boys and two girls. Laura disagreed, of course. She claimed there was a better chance of three children of one gender and one of the other.

Over lunch, their debate grew hotter and hotter, finally resulting in a bet. Laura and Steve declared the loser of the bet would have to stand up on a chair and cluck like a chicken during lunch. They went to see their favorite teacher, Mr. Hansen, hoping he would know the answer. He suggested they list all the possible outcomes in a table to figure it out, or draw a diagram.

With this good advice, Laura and Steve both solved the problem later that day. One of them was the lucky winner, and the other was the "clucky" winner.

a) What is the probability of 2 boys and 2 girls? _______________

b) What is the probability of 3 of one and 1 of the other? _______________

9) Mental Math: do these in your head, and write down the answers. When you're done *check your answer* with pencil and paper, or calculator, or a friend.

a) What is two-thirds of 66?

b) What is the first and last third of your name?

c) What are all the factors of the number 66?
Please include all the composite factors as well as the prime factors.

d) Round the number 2252 to the nearest hundred.

e) What are the next two numbers in this series: 24, 17, 10, 3, ____, _____

f) When you convert 73 ½ % to a decimal, is the result 73.5, 0.0735 or 0.735?

g) When you convert 2 ½ % to a decimal, what is the result?

h) Work backward to solve this problem.

The Backward Boy eats lunch in the middle of the day, just like everyone else.
But he eats dinner in the morning and breakfast at night!
Today, he ate 3 more Backward Burgers than tuna fish sandwiches.
He ate 4 less tuna fish sandwiches than bowls of cereal.
The boy ate 6 bowls of cereal for dinner.
How many Backward Burgers did the Backward Boy eat?

i) Did you check your answers? If not, go back and check your answers now.
Hint: It's okay to ask someone to check your answers for you!

You're done! Detach the homework from the lesson, and turn in just the *homework*.