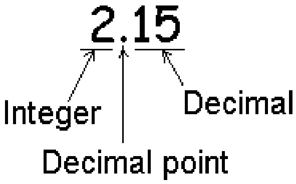
Are you tired of finding common denominators to add fractions?
Are you tired of converting mixed fractions into improper fractions, just to multiply and convert them back?
Are you tired of reducing fractions to their simplest form?
Do you wish there was an easier way? Well, thanks to Math Club *now there is!*

Decimal Numbers

Decimal numbers make it easy to work with fractional amounts. We show fractions of dollars using the decimal system. We can make distance calculations easier by using the metric system instead of miles-feet-inches.

Decimal numbers or *decimal fractions* are a proper fraction based on the number 10. They use a decimal point separating the fraction from the whole number. Decimal numbers are written without showing denominators. Decimal numbers have three parts: an integer, a decimal point, and a decimal number.



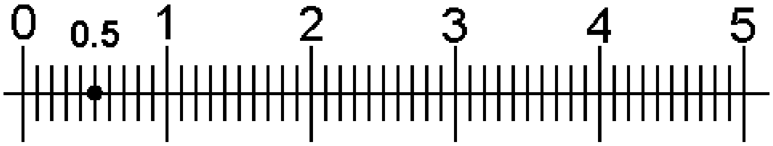
We use the same place value system for decimals that we use for whole numbers. Anywhere you look in a whole number, the value of the place to the right of a digit is always ten times smaller. The tens' place is ten times smaller than the hundreds' place, and the ones' place is ten times smaller than the tens' place.

To find the value of the place to the right of the ones' place, divide one by ten: 1 ÷ 10 = 1/10 = one-tenth. The first place to the right of the decimal point is the **tenths**' place. The value to the right of the tenths' place is the 1/10 ÷ 10, and is called the **hundredths'** place. This chart shows some common place values.

4,	3	2	1	.	1	2	3
Thousands	Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths

Writing Decimal Numbers

Between each whole number, this number line is divided into tenths. Decimals with values less than 1 are written with a 0 in the ones' place.

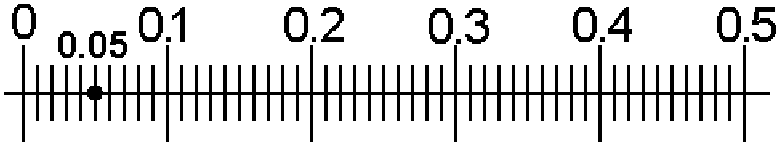


To write *five tenths*, write a **0** in the ones' place and a **5** in the tenths' place, with a decimal point in between: 0.5.

Whole numbers can also be written with one or more zeros after the decimal point:

3 = 3.0, 1 = 1.000, 0 = 0.00000

The number line below shows what is in the area between 0 and 0.5 from the number line above. Between each of the tenths' values, the number line is divided into hundredths. Notice the denominator, hundredths, names the last place on the right that is holding a digit.

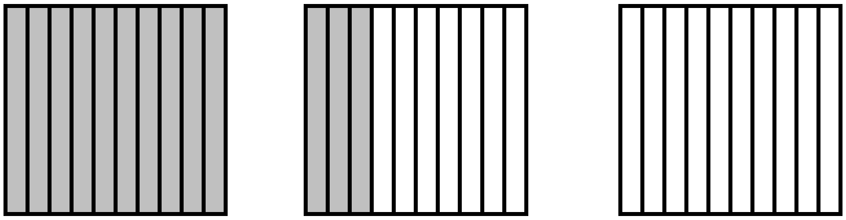


To write *five hundredths*, write a 0 in the tenths place and a 5 in the hundredths' place: 0.05. The 0 is needed as a placeholder between the decimal point and the hundredths' digit.

Example: How would you write *three and nine hundredths* as a decimal?

Solution: The *and* tells you where to put the decimal point: after the 3. The *hundredths* tells you there are two decimal places in the number and that a 9 goes in the hundredths' place: 3.**09**

Example: Dylan's backyard is fenced in on three sides. Each side of the fence is made up of ten boards. (It's a small yard!) Dylan agreed with his dad to paint one half of the fence. So far Dylan has painted **one side** of the fence and **three boards** on another side.
What decimal number represents the amount of fence Dylan has painted?



Solution: We know that Dylan has painted one side (10 boards) plus 3 boards out of 10 on another side.

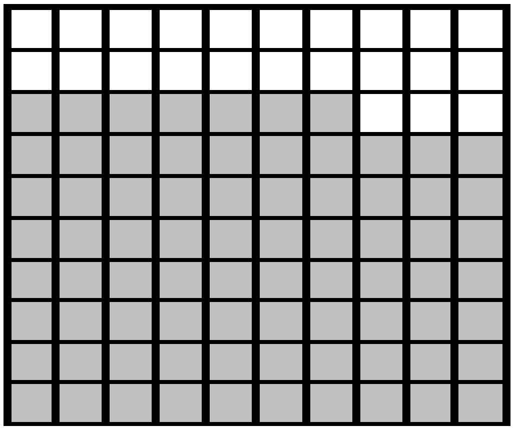
10 boards = 1 side

3 boards out of 10 = $^{3}\!/_{10}$ of a side

Adding 1 plus 3/10 we get the mixed number 1 $^{3}\!/_{10}$, which we write as the decimal **1.3**.

We say "one and three tenths."

Example: Esther's mom is re-shingling the roof of their house. It takes 100 shingles to complete the roof. What decimal represents the amount of roof Esther's mom has finished if she only has 23 shingles left?



We know the whole roof is 10 rows of 10 shingles, which is 10 times 10 = 100 shingles.

We also know Esther's mom has 23 shingles left, so she has used: 100 - 23 = _______

So Esther's mom has completed _____/100 of the roof. We write this fraction as the decimal **0.77**. We say "seventy seven hundredths."

Changing Decimals to Fractions

It is easy to convert decimals to fractions. Look at the number 0.27, or twenty-seven hundredths. The *hundredths* tells you the denominator is 100. The numerator will be the numeral to the right of the decimal point, which is 27 in this case. Do not write the decimal point in the fraction.

$$0.27 = \frac{27}{100}$$

You must look for fractions that can be reduced. For example, look at the number 0.32, or thirty-two hundredths. Write the fraction and then reduce it to lowest terms.

$$0.32 = \frac{32}{100} = \frac{16}{50} = \frac{8}{25}$$

Changing Fractions to Decimals

Decimal numbers are really just a shortcut method of writing common fractions, where the denominator is 10 or 100 or 1000 and so forth. You can convert fractions into decimal numbers by making the denominator a multiple of 10. The denominator will indicate the smallest place value that should appear in the decimal.

$$\frac{5}{10} = 0.5 \qquad \frac{13}{100} = 0.13$$

$$\frac{8}{100} = 0.08 \qquad \frac{19}{10} = 1.9$$

You can also change a fraction to a decimal if the fraction has a denominator other than 10, but you need to know about decimal division to do this. Or, use a calculator.

Note: When a decimal number is less than one, we always write the zero to the left of the decimal point. Without the leading zero, the reader *might not* notice that tiny little mark for the decimal point.

Repeating Decimals

Some fractions when written as a decimal number will repeat forever. To write a repeating decimal, put a bar over the rightmost digit (or digits) that repeat. For example, $\frac{1}{11} = 0.09\overline{09}$ or simply $0.\overline{09}$.

Why do some fractions repeat and others don't? Notice that decimal numbers represent the addition of $\frac{1}{10}$ and $\frac{1}{100}$ and $\frac{1}{1000}$ and so forth. Now take a closer look at the factors in these fractions: $\frac{1}{10}=\frac{1}{2\times 5}$ and $\frac{1}{100}=\frac{1}{2\times 5\times 2\times 5}$. In fact, every position in the decimal number represents some multiple of 2's and 5's. This works well for fractions containing any multiples of halves and fifths. But it is impossible to *exactly* represent thirds, sixths or anything with factors other than 2 or 5!

Addition and Subtraction

Example: David wants to dance all night at the PTA Sock Hop on November 5th at Sunny Hills. To get in shape for the event he plans to walk 10 miles. So far, he has walked **2** miles on Monday, **1.7** miles on Tuesday, **3.56** miles on Wednesday and **1.24** miles on Thursday. (Don’t ask me how he measured it.) How many miles will he have to walk on Friday to make a total of **10** miles?

Solution: We know David has walked 2, 1.7, 3.56 and 1.24 miles already. To get the total so far we add them.

To add, write the numbers in a column aligning the values and decimals. Add each column from right to left and carry down the decimal point:

$$\begin{array}{r} 2.00 \\ 1.70 \\ 3.56 \\ +1.24 \\ \hline \end{array}$$

miles so far

Total miles David wants to walk = 10. Subtract the total so far from his goal:

$$\begin{array}{r} 10.00 \\ -8.50 \\ \hline \end{array}$$

miles to walk on Friday

To subtract, write the numbers in a column aligning the values and decimals. *Hint: Neatness counts!*

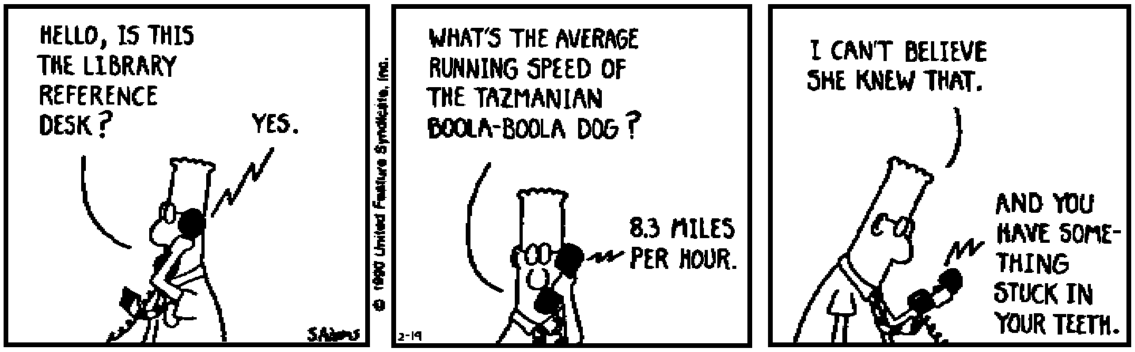
Padding Decimals with Zeros

If a number is “longer” than the others, pad them with zeroes so they are all the same length. Do you remember *why* it is that you can pad with zeroes and the value doesn’t change? It is because zero is the identity element for addition.

Vocabulary

- *Decimal form* - a list of digits that represent a fraction by using a decimal point. Most hand-held calculators always show fractions in decimal form. For example, decimal $0.1 = {}^{1}\!/_{10}$ and $0.12 = {}^{12}\!/_{100}$ and $0.003 = {}^{3}\!/_{1000}$.
- *Decimated form* – a severely sliced-up object, such as a tasty pumpkin pie ready to eat. Comes from the root word “deci-“ meaning ten, and therefore implying the object is sliced into at least ten pieces.
- *Decimal fractions* – the proper name for these numbers is *decimal fractions*, since it is used in all civilized countries of the world except for the USA. However, the common usage here is *decimal numbers*, so that’s what this lesson uses, too.
- *Recursive* - see recursive

Dilbert by Scott Adams



1) Re-write the fractions and mixed numbers in their decimal equivalent. Remember, neatness counts!

a) $\frac{3}{10}$ *Example: 0.3*

b) $\frac{45}{100}$

c) $\frac{57}{100}$

d) $3\frac{1}{10}$

e) $3\frac{14}{100}$

f) $3\frac{141}{1000}$

g) $3\frac{1416}{10000}$

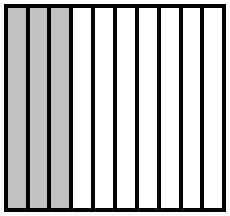
h) $1\frac{5}{100}$

i) $1\frac{5}{1000}$

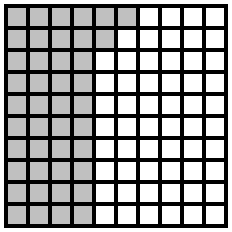
j) $99\frac{99}{100}$

2) Write the decimal number that is equivalent to each shaded-in area.

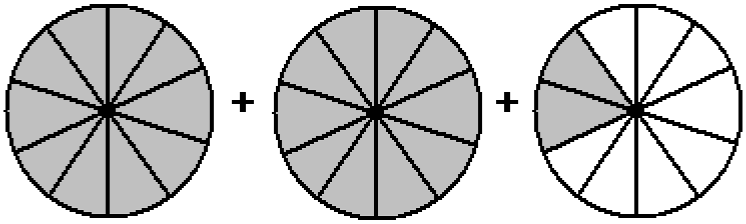
a)



b)



c)



3) Write the decimal number for:

a) fifty-six tenths *Example:* $\frac{56}{10} = 5.6$

b) fifty-seven hundredths

c) nine and nine tenths

d) twelve tenths

e) sixty-six and sixty-seven hundredths

4) Convert the decimals to a common fraction and reduce to lowest terms.

a) 0.25 *Example:* $\frac{25}{100} = \frac{5}{20} = \frac{1}{4}$

b) 0.4

c) 0.125

d) 0.375

e) 0.625

f) 0.5

g) 0.05

h) 0.005

5) Compute the following decimals.

a) $$\begin{array}{r} 6.58 \\ \underline{+2.42} \end{array}$$

b) $$\begin{array}{r} 9.34 \\ \underline{-3.06} \end{array}$$

c) $$\begin{array}{r} 45.7 \\ \underline{+92.15} \end{array}$$

d) $$\begin{array}{l} 54.655 \\ \underline{-1.92\ \ } \end{array}$$

e) $$\begin{array}{r} 518.051 \\ \underline{+36.007} \end{array}$$

f) $$\begin{array}{l} 22.1 \\ \underline{-9.7155} \end{array}$$

6) Write these problems in columns, aligning the values and decimals. Use a separate sheet of paper if needed. Then compute the following:

a) 19.3 + 47.22 =

b) 23.8 + 12.95 =

c) 29.838 - 0.425 =

d) 217.19 + 71.3 + 35 =

7) If you write these fractions as decimal numbers, will they be a repeating decimal? Remember: it is repeating if the denominator has any prime factor other than 2 or 5. Circle *repeating* or *nonrepeating*.

a) $1/3$ *nonrepeating* [✓ *repeating*] *(Example)*

b) $1/4$ *nonrepeating* *repeating*

c) $1/7$ *nonrepeating* *repeating*

d) $1/9$ *nonrepeating* *repeating*

e) $1/25$ *nonrepeating* *repeating*

8) Write these problems in columns, aligning the values and decimals. Then compute the following:

a) A male tiger weighs 226.9 pounds. A female tiger weighs 37.3 pounds less. How much does the female weigh?

b) The maximum weight that a container can hold is 39.55 pounds. If Hannah put two items, one weighing 10.24 pounds and the other weighing 17.88 pounds, into the container, how much more weight can the container hold?

9) Mental Math. Do these in your head, and then *check your answer* with pencil and paper, or with a calculator.

a) Start with 489, subtract 39, then divide by 50.

b) What is $2\frac{1}{6}$ times twelve?

c) What is your name?

d) What is the result of $16 \div 2 \times 4 \div 8$?

e) Name the prime factors of 10.

f) What is the least common multiple (LCM) of 4 and 6?

g) What is the greatest common factor (GCF) of 4 and 6?

h) Did you check your work? Circle *yes* or *no*.

i) Did you circle *no*? Go back and check your work!
You may use a parent, a friend, a calculator, or a friend of your parent's calculator.