

Multiplying And Dividing Fractions

Fractions are used when we don't have a whole number of something. Although dogs and horses always come in whole numbers, there are *many* things that come in parts. For example, measurements of baking ingredients and distance commonly include fractions. When you look at averages you usually get fractional parts, even for things that come as units. When you combine fractions together, you often need to multiply and divide fractions.

Multiplication

To **multiply** fractions, convert all mixed numbers to improper fractions, and write them on a common bar.

$$\frac{\text{multiply the numerators}}{\text{multiply the denominators}} = \text{Product (or new fraction)}$$

Example: $\frac{3}{4} \times \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$

To reduce your labor, cancel factors that appear in both numerator and denominator. That is, if the same number appears in both the top and bottom then you can cross out the pair. This keeps the numbers smaller and makes them easier to use.

Example: $\frac{5}{18} \times \frac{18}{39} = \frac{5 \times 18}{18 \times 39} = \frac{5}{39}$

Identity Element of Multiplication

Austin said $\frac{9}{12}$ of his toy cars were red, but Ben thought $\frac{3}{4}$ of them were red. The teacher said both boys were correct. Why are they both right?

They are both right because $\frac{9}{12}$ and $\frac{3}{4}$ are equivalent fractions.

To understand this, we need to consider the **identity element** of multiplication:
Whenever the same non-zero number appears in both the numerator and denominator, the fraction is equal to 1.

Thus, $\frac{2}{2} = 1$, $\frac{3}{3} = 1$, and even $\frac{99}{99} = 1$.



The number 1 is the identity element of multiplication. That means whenever we multiply a number by 1, we get the same number.

Think: What is the identity element of division? _____

To prove that $\frac{3}{4}$ and $\frac{9}{12}$ are equivalent fractions, we will show how $\frac{3}{4}$ has been multiplied by 1 to get the new fraction:

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

The identity element can be written in many forms. Note that anything (except zero) divided by itself is 1, so any of these forms could be used for the identity element of multiplication:

$$\frac{5}{5} \quad \frac{1/3}{1/3} \quad \frac{-1}{-1} \quad \frac{\pi}{\pi} \quad \frac{x}{x}$$

Choose the form of the identity element that simplifies your calculations.

One is the **identity element for multiplication** of numbers, since $a \times 1 = 1 \times a = a$ for all values of a .

Multiplying Mixed Fractions

To multiply mixed fractions, first you convert them into improper fractions and then multiply.

Example: $2\frac{1}{4} \times 5\frac{1}{2} = ?$

$$2\frac{1}{4} = \frac{(2 \times 4) + 1}{4} = \frac{9}{4}$$

$$5\frac{1}{2} = \frac{(5 \times 2) + 1}{2} = \frac{11}{2}$$

$$\frac{9}{4} \times \frac{11}{2} = \frac{9 \times 11}{4 \times 2} = \frac{99}{8} = 12\frac{3}{8}$$

Again you can minimize your work by canceling factors before you do the multiplication.

Dividing by Fractions

To divide, we multiply the dividend (top) by the *reciprocal* of the divisor (bottom).

Example: $\frac{7}{8} \div \frac{3}{4}$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$, therefore:

$$\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3} = \frac{28}{24} = 1 \frac{4}{24} = 1 \frac{1}{6}$$

So *why* does it work to multiply by the reciprocal of the divisor (bottom)? It all comes from the identify element of multiplication again. Here's the reasoning behind it... Start by writing the expression as one fraction above another:

$$\frac{\left(\frac{7}{8}\right)}{\left(\frac{3}{4}\right)}$$

Now simplify the big fraction by multiplying by the identity element, one. But in this case we'll write the number "one" as " $\frac{4}{3}$ divided by $\frac{4}{3}$ ":

$$\frac{\left(\frac{7}{8}\right) \left(\frac{4}{3}\right)}{\left(\frac{3}{4}\right) \left(\frac{4}{3}\right)}$$



Now we can multiply out the top and bottom of the big fraction:

$$\frac{\left(\frac{7}{8} \times \frac{4}{3}\right)}{\left(\frac{3}{4} \times \frac{4}{3}\right)} = \frac{\frac{7}{8} \times \frac{4}{3}}{1} = \frac{7}{8} \times \frac{4}{3}$$

This shows that dividing something by a fraction is the *same* as multiplying something by the reciprocal of the fraction.

Questions!

When we *add* fractions, we find a common denominator and add the numerators.

When we *multiply* fractions, we multiply numerators together and denominators together, with no regard to commonality.

1. Why do we not have to find a common denominator when multiplying?
2. Why do we multiply both numerators and denominators?

Answers!

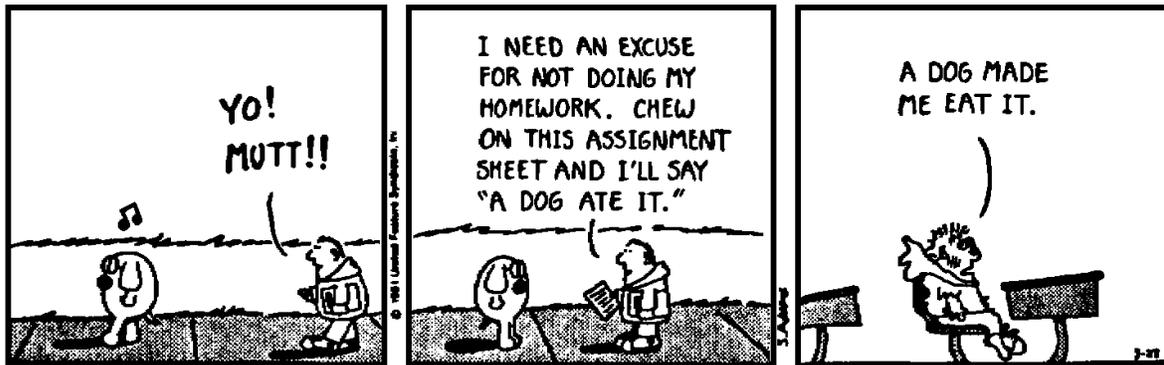
1. The general rule for *adding* is: **You must add like quantities.** You can't add "thirds" and "fifths" for the same reason you can't add "apples" and "elephants" -- it just doesn't make sense. But the same fraction has many different names so, unlike apples and elephants, you can change "thirds" and "fifths" to their equals that happen to have a common name such as "fifteenths" or "thirtieths".
2. The general rule for *multiplying* is: **If you want the product to make numerical sense you must do what the numbers tell you to do.** For example: "Two-thirds" of X means divide X into three equal pieces and take two of those pieces. If X happens to be $\frac{5}{6}$ the instruction "two thirds of X" means $(\frac{2}{3}) * (\frac{5}{6})$. So take your "five-sixths" and divide it into three equal pieces, each of size "five-eighteenths". Then 2 sets of 5 pieces makes 10 "eighteenths" or $\frac{10}{18}$.

Vocabulary

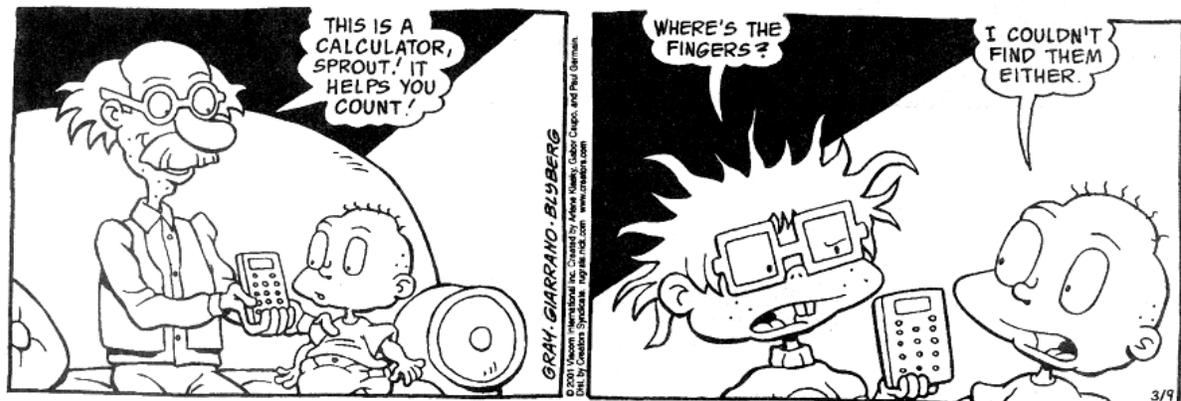
- *Improper fractions* - a fraction where the numerator (top) is equal or larger than the denominator (bottom). For example, $\frac{10}{10}$, or $\frac{17}{16}$, or $\frac{5}{2}$.
- *Proper fractions* - a fraction where the numerator (top) is less than the denominator (bottom).
- *Identity element* - the number you can apply which does not change the result. The identity element of multiplication and division is 1. The identity element of addition and subtraction is zero.
- *Mixed numbers* - a whole number and a fraction. For example, $7\frac{1}{2}$.
- *Numerator* - the number in a fraction which appears above the line.
- *Denominator* - the bottom part of a fraction, which indicates the number of parts into which the whole unit is divided
- *Dividend* - the number to be divided (same as numerator)

- *Divisor* - the number by which something is to be divided (same as denominator)
- *Reciprocal* - the number you would use to multiply to get an answer of '1'.
For example, the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$, because $\frac{3}{2} \times \frac{2}{3} = 1$. Note that plain whole numbers do have reciprocals, for example, the reciprocal of 2 is $\frac{1}{2}$. You can get the reciprocal of a fraction by swapping the numerator and denominator. Reciprocals are especially useful in dividing fractions, because dividing a fraction into a number is the same as multiplying by the reciprocal of the fraction.
- *Reciprocating engine* - an engine that works mainly by parts that alternately move forward and back (or up and down). This is not the same as either a *reciprocal* or a *recipe*!

Dilbert by Scott Adams



Rugrats by Nickleodeon



Multiplying and Dividing Fractions

- 1) Multiply the following fractions. Show your answers as proper fractions, that is, make the numerator (top) less than the denominator (bottom). Always reduce fractions!

a) $\frac{6}{7} \times \frac{5}{4} =$ *Example:* $\frac{6}{7} \times \frac{5}{4} = \frac{6 \times 5}{7 \times 4} = \frac{30}{28} = \frac{15}{14} = 1\frac{1}{14}$

b) $\frac{1}{4} \times \frac{1}{5} =$

c) $\frac{7}{10} \times \frac{1}{2} =$

d) $\frac{5}{7} \times \frac{7}{5} =$

e) $\frac{12}{5} \times \frac{5}{3} =$

f) $\frac{5}{8} \times \frac{4}{3} =$

g) $\frac{12}{7} \times \frac{21}{24} =$

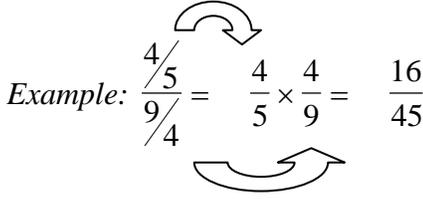
h) $\frac{3}{7} \times \frac{7}{3} =$

i) $\frac{6}{5} \times \frac{4}{3} =$

- 2) Divide the following fractions. Show your answer as a proper fraction. Show your work.

a) $\frac{4}{5} \div \frac{9}{4} =$

Example: $\frac{4}{9} \div \frac{5}{4} = \frac{4}{9} \times \frac{4}{5} = \frac{16}{45}$



b) $\frac{3}{4} \div \frac{8}{3} =$

c) $\frac{4}{7} \div \frac{2}{7} =$

d) $\frac{9}{11} \div \frac{2}{22} =$

e) $\frac{11}{12} \div \frac{5}{6} =$

f) $\frac{2}{9} \div \frac{5}{3} =$

g) $\frac{7}{9} \div \frac{1}{8} =$

h) $\frac{3}{5} \div \frac{3}{5} =$

i) $\frac{1}{2} \div \frac{3}{4} =$

- 3) Divide (or multiply) the following mixed fractions. Read carefully! Show your answers as mixed numbers or proper fractions. Show your work.

a) $1\frac{5}{9} \times \frac{3}{4} =$ *Example:* $\frac{14}{9} \times \frac{3}{4} = \frac{42}{36} = \frac{21}{18} = \frac{7}{6} = 1\frac{1}{6}$

b) $1\frac{2}{3} \div \frac{1}{6} =$

c) $1\frac{5}{9} \div 1\frac{4}{5} =$

d) $1\frac{5}{6} \times 2\frac{1}{3} =$

e) $2\frac{1}{2} \div 1\frac{1}{2} =$

f) $1\frac{1}{5} \times \frac{1}{2} =$

g) $1\frac{1}{4} \times 3\frac{3}{4} =$

h) $7\frac{1}{2} \div 2\frac{1}{2} =$

- 4) Solve the following problems. Decide if you need to multiply or divide the fractions. Write all answers as a proper fraction.
- a) Rita the Rwandan Ruler Reader reads rulers all day long. The rulers measure fractions of an inch, and each measurement is a multiple of $\frac{1}{16}$ of an inch. Write the sixteen possible measurements $\frac{1}{16}$ to 1 inch, reducing fractions for each one.
- b) Have you ever eaten M&M's? Each 1.69-ounce bag of M&M's is $\frac{2}{19}$ of a pound. (Really!) If you bought a single large $1\frac{1}{2}$ -pound bag and divided it into portions of $\frac{2}{19}$ pounds each, how many portions would you have?
- c) I have $2\frac{2}{3}$ cups of sugar. Hillary's fabulous Smoothie recipe takes $\frac{1}{3}$ cup of sugar for each one. How many Smoothies can Hillary make?
- d) Suppose I am growing a pumpkin for Halloween. Right now it weighs 25 pounds. Each week it grows by 20% compared to the previous week. That is, it becomes $1\frac{1}{5}$ times the size of the week before. How many pounds will it be after growing for three weeks?

- 5) Mental Math: do these in your head, and write down the answers. When you're done, *check your answer* with pencil and paper, or with a calculator or parent.
- a) Start with 38, then divide by 5.
 - b) Start with the number of tens in 54, and multiply by 12.
 - c) What is 11 multiplied by 11?
What is 12 multiplied by 11?
 - d) Add $\frac{1}{3} + \frac{1}{12} =$
 - e) What is your name?
 - f) Divide the fraction $\frac{4}{5}$ by three.
 - g) Gabriel D. Fahrenheit was born in 1686. How old was he when he invented the mercury thermometer in 1714?
 - h) Blaise Pascal was born in 1623. He invented a calculating machine in 1642, some principles of which were used in the first desktop calculators. How old was he in 1642?
 - i) Renee Descartes was born in 1596 in France. (He would be very old today!) He invented new ways to work with the two-dimensional coordinate system that was published in 1638.
 - * How old was he then?
 - * How old would he be today?
 - j) Answer *yes* or *no*: Did you check your work? Great!

Separate this *homework* from the lesson, and turn in only the homework.