

SHS Math Club
2009-10
Math Toolbox

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I. Sequences

a. Arithmetic Sequence

- i. Definition= in an arithmetic series, the difference between terms is constant. Ex: $4 + 10 + 16 + 22\dots$ they all have a difference of six between terms.
- ii. Formula for sum of terms in sequence= $(1/2) * n * (a+z)$
 1. n = number of terms in the sequence
 2. a = first term in sequence
 3. z = last term in sequence
- iii. Answer to example if the sequence goes up to 100 is : $(1/2) x 17(4+100) = 884$

b. Geometric Sequence

- i. Definition= it is a sequence such that each successive term is obtained from the previous term by multiplying a fixed number called a ratio. Ex: $3 + 6 + 12 + 24 + 48 + 96$
- ii. Formula for sum of terms in sequence
 1. *Finite Geometric Sequence* = $(a(1-r^n))/(1-r)$
 - a. a = first term in sequence
 - b. r = common ratio
 - c. n = number of terms
 - d. note: r does not equal 0
 2. *Infinite Geometric Sequence* = $a/(1-r)$
 - a. If the absolute value of r is greater than or equal to one, then the infinite series does not have a sum

c. Figurative Number Sequences

- i. *Triangular numbers*
 1. Equation = $n(n+1)/2$.
 2. First terms of sequence= 1, 3, 6, 10, 15, 21, 28
 3. n = order of number in sequence. Ex: if $n = 3$, then look for the third term in the sequence $(3(4)/2) = 6$.
- ii. *Square numbers*
 1. Equation = n^2
 2. First terms of sequence = 1, 4, 9, 16, 25, 36, 49
 3. n = order of number in sequence.
 4. Try this: What is the sum of the first “ n ” odd positive numbers??
- iii. *Pentagonal numbers + beyond* = $n(3n-1)/2, n(4n-2)/2, n(5n-3)/2\dots$

II. Geometric Formulas

a. The Pythagorean Theorem

- i. $a^2 + b^2 = c^2$
- ii. only works when “a” and “b” are the lengths of the legs of a right triangle, and “c” is the hypotenuse
- iii. Pythagorean triples:
 1. 3, 4, 5 $(3^2 + 4^2 = 5^2)$ $(9 + 16 = 25)$
 2. 8, 15, 17 $(8^2 + 15^2 = 17^2)$ $(64 + 225 = 289)$
 3. 5, 12, 13
 4. 7, 24, 25
 5. 20, 21, 29
 6. 12, 35, 37
 7. 9, 40, 41
 8. 11, 60, 61
 9. Note: If you double “a” and “b”, and “c”, you can get a Pythagorean triple that is also the same as the old one (ex: 3, 4, 5 double = 6, 8, 10 $(36 + 64 = 100)$). Those triples are not repeat listed here.

b. Triangle Formulas (not including Pythagorean)

- i. The 45-45-90 degree triangle
 1. Properties are:
 - a. It is half a square
 - b. Hypotenuse = $S * \sqrt{2}$ if S is the side length of the leg
 - ii. The 30-60-90 degree triangle
 1. Properties are:
 - a. Half of an equilateral triangle
 - b. If shortest leg = S, then hypotenuse = $2S$, and the long leg = $S\sqrt{3}$
 - iii. Sine, Cosine, and Tangent
 1. Sine of an angle = opposite leg / hypotenuse
 2. Cosine of an angle = adjacent leg / hypotenuse
 3. Tangent of an angle = opposite leg / adjacent leg
 4. $\sin^2 x + \cos^2 x = 1$
 5. $\tan(x) = \sin(x) / \cos(x)$
 6. $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$
 7. $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$
 8. $\tan(x+y) = (\tan(x) + \tan(y)) / (1 + \tan(x)\tan(y))$
 9. Note: you are not expected to memorize 6, 7, 8 they are just for fun, and 1, 2, 3 ONLY work in a right triangle!!!
 10. Many... many... more
 - iv. Heron's Formula
 1. Area of triangle = $\sqrt{(sp) * (sp - a) * (sp - b) * (sp - c)}$
 - a. Sp= semi-perimeter
 - b. a= first side, b= second side, c = third side
 - c. Works for any triangle

- c. Properties of Polygons
 - i. Number of diagonals in a polygon with “n” sides
 - 1. $n(n-3)/2$
 - ii. Number of degrees in an n-sided polygon
 - 1. $180(n-2)$
 - iii. Sum of the exterior angles of any polygon
 - 1. 360 degrees

III. Miscellaneous Geometry

- a. Space Diagonal of a cube : $S * \sqrt{3}$ where S is one length of the cube
- b. Areas and Volumes
 - i. Area of Square = s^2
 - ii. Area of Square = $(d^2)/2$
 - 1. d = diagonal
 - iii. Area of a rhombus $(d_1 * d_2)/2$
 - 1. d1 and d2 are the diagonals
 - iv. Area of triangle = $\frac{1}{2} * b * h$
 - 1. b=base, h=height
 - v. Area of triangle = $\frac{1}{2} * b * c * \sin(A)$
 - 1. b= adjacent side 1, c = adjacent side 2, and A = the angle for which the sides b,c are adjacent to
 - vi. Area of circle = $(\pi) * r^2$
 - 1. r= radius
 - vii. Area of trapezoid = $\frac{1}{2} * h(b_1 + b_2)$
 - 1. h = height, $b_1; b_2$ are the two bases
 - viii. Volume of cylinder= $B * h$
 - 1. B= area of base and h= height
 - ix. Volume of cone= $(1/3) * B * h$
 - x. Volume of sphere = $(4/3) * (\pi) * r^3$
 - xi. Surface area of sphere = $4 * (\pi) * r^2$

IV. Factoring Method

- a. Factor out any common factors
- b. If it is two terms, check whether it can be rewritten as the difference of two cubes, or the sum of two cubes
- c. If it is three terms, check for the perfect square trinomials
- d. If it has more than three terms, try to factor by grouping

V. **Combinations/Permutations**

- a. Number of permutations of “n” different things:
 - i. $n!$
- b. Number of permutations of “n” things where “r” things are the same:
 - i. $n!/r!$
- c. Number of permutations of “n” things taken “r” at a time:
 - i. $n!/(n-r)!$
- d. Number of combinations of “n” things taken “r” at a time:
 - i. $n!/(r! * (n-r)!)$
 - ii.

VI. **List of basic squares/cubes**

Number (n)	Square (n^2)	Cube (n^3)
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728

VII. **List of Factorials**

- a. Expressed as “n!”
- b. $n! = n * (n-1) * (n-2) * (n-3) \dots * 3 * 2 * 1$

Number (n)	Factorial (n!)
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800